Worcester County Mathematics League

WOCOMAL Varsity Meet #1

Coaches' Booklet

October 13, 2004

Round 1:   Arithmetic (NO CALCULATORS)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1.   Evaluate:

$$2-(1-(2-(1-(2(1-2)))))$$

2.   If $a * b = \max(a,b) = $ larger of a and b, and $a^b = \min(a,b) = $ smaller of a and b, evaluate:

$$[-((3*2)^2)]^3$$

3. Rewrite the following expression, using exactly two sets of parentheses, so that the following expression, when evaluated, equals 20: (note: x means multiplication in this problem)

$$2 \text{ x } 8 \div 2^2 - 3 + 3 \text{ x } 36 \div 12$$

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. $\underline{2 \text{ x } 8 \div 2^2 - 3 + 3 \text{ x } 36 \div 12}$

Hudson, St. John's, Bromfield

Round 2:   Algebra 1 (open)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1.   Twice the sum of three consecutive multiples of 15 is 270. What are the three multiples of 15?

2.   The sum of the square roots of two numbers is 5. The two numbers differ by 5. What are the numbers?

3.   Solve for x and y in terms of a and b where $a \neq b$.

$$bx + ay = -2a + b$$

$$b^2x + a^2y = -ab$$

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. x = _____ y = _____

Shepherd Hill, Burncoat, West Boylston

Round 3:   Set Theory

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. If A is a set, then A' is considered the complement of A. Let U = {1,2,3,4,5,6,7,8}, where U is considered the Universal set (for this problem). Specify by roster:

$$\{\{3,5,1,4\} \cap U\}'$$

2. J = set of integers. A' means the complement of A in J. A = {xεJ/x>10}; B={xεJ/5<x<15}; C = {xεJ/x<-7}. Using set notation, as in the preceding sentence, what is A' ∩ B' ∩ C' ?

3. Twenty-four dogs are in a kennel. Twelve of the dogs are black, six of the dogs have short tails, and fifteen of the dogs have long hair. There is only one dog that is black with a short tail and long hair. Two of the dogs are black with short tails and do not have long hair. Two of the dogs have short tails and long hair but are not black. If all of the dogs in the kennel have at least one of the mentioned characteristics, how many dogs are black with long hair but do not have short tails?

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. {xεJ/_____

(3 pts.)   3. _____

Tantasqua, Quaboag

Round 4:   Measurement

ALL ANSWERS MUST BE AS DIRECTED IN THE PROBLEM. RADICALS MUST BE WRITTEN IN SIMPLEST RADICAL FORM.

1.   Determine the **exact** area of a regular hexagon whose side measures 7 .

2.   A rectangle has dimensions of $(x + 2)$ and $(3x + 3)$. Determine the **exact** value of x so that the perimeter and area of the rectangle will have the same numerical value.

3.   A cube and a sphere would have the same volume if the sphere's radius were halved. What is the **exact** ratio of the sphere's radius to the cube's side?

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Shrewsbury, Assabet

Round 5:   Polynomial Equations (NO CALCULATORS)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1.   Solve: $x^3 - 6x^2 + 11x - 6 = 0$

2.   Suppose a cubic polynomial has roots of $\{1, -1, 2\}$ and has a value of 8 when
        $x = 3$. Determine the value of the polynomial when $x = 0$.

3.   Determine the sum of all possible integers **N** such that $x^3 + 2x^2 + Nx - 3 = 0$ has
        at least one rational root.

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Southbridge, Hudson, Tantasqua

TEAM ROUND

**ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THE
SEPARATE TEAM ANSWER SHEET.   (2 points each)**

1.   If the operation x * y is defined as x*y = xy – 3 + x – 3y, determine the value of
        2* (4*6).

2.   Determine an equation in a and b of smallest degree which has all ordered pairs
        (a,b) listed as solutions (5,-3), (12,-17), (1,5),(4,-1), and (-2,11). Write your
        answer in the form of: "b = an expression involving a".

3.   Factor completely over the set of integers: $10x^{4k+6} – 7x^{2k+3} -12$.

4.   If the equation of the line passing through the point (0,-5) and perpendicular to the
        graph of y = ¾ x + 2 is **a**x + **b**y = **c**, where **a** and **b** are relatively prime and **a**>0,
        what is the value of the sum (**a** + **b** + **c**) ?

5.   Determine the **exact** volume of a right circular cone whose diameter is 20 cm and
        whose slant height is 26 cm.

6.   If the solutions to the system of equations $x^2 + y^2 = 13$ and 2x – 3y = -5 are
        (**a,b**) and (**c,d**), then determine the value of **a** + **b** + **c** + **d**. Write your answer as
        a fraction (**N/D)** where **N** and **D** are relatively prime.

7.   A stew was cooked in a cylindrical pot 24 cm in diameter and 20 cm deep. After
        one meal, there remained 6 cm of stew in the pot. When this was transferred to a
        smaller pot 18 cm in diameter, how deep was the stew to the nearest tenth of a
        cm?

8.   Let S = {1,4,9,16,25,…} be the set of squares of positive integers. Find the
        square **t** such that (**t + 43**) is also a member of S.

9.   Given that 2 + √3 is one of the solutions of the equation

        $x^4 – 14x^3 + 54x^2 – 62x + 13 = 0$, find the **exact sum** of the other three solutions.

 Southbridge, Hudson, Tantasqua, Leicester, Mass Academy, Algonquin, Notre Dame
Academy, Shrewsbury, Clinton, and Worcester Academy

Round 1: Arithmetic

1. (1 pt.)   **0**

2. (2 pts.)   **-2**

3. (3 pts.) $2(8 \div 2)^2 -(3 + 3 \text{ x } 36 \div 12)$

Round 2: Algebra-open

1.   (1 pt.)   30,45,60

2.   (2 pts.)   4 and 9

3.   (3 pts.) x = -2a/b ; y = b/a

Round 3: Set Theory

1.   (1 pt.)   {2,6,7,8}

2.   (2 pts.) {xεJ/ -7 ≤x ≤5}
            or   {xεJ/ -8 <x <6}
3.   (3 pts.)   3

Round 4: Measurement

1.   (1 pt.)   (147√3)/2

2.   (2 pts.)   1

3.   (3 pts.) $\sqrt{(6\pi^2)}/\pi$

Round 5: Polynomial Equations

1.   (1 pt.)   1,2,3

2.   (2 pts.)   2

3.   (3 pts.)   -20

TEAM ROUND (2 pts. Each)

1.   -8

2. $b = -2a + 7$

3. $(5x^{2k+3} + 4)(2x^{2k+3} - 3)$

4.   -8

5. $800\pi \text{ cm}^3$

6.   10/13

7.   10.7 cm

8. $t = 441$

9. $12 - \sqrt{3}$

October 13, 2004　　　　　　**WOCOMAL Varsity Meet**

TEAM ROUND　　　　　　**School: _____**

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THIS SEPARATE TEAM ANSWER SHEET.　(2 points each)

　　1.　　_____

　　2.　　_____

　　3.　　_____

　　4.　　_____

　　5.　　_____

　　6.　　_____

　　7.　　_____

　　8.　　_____

　　9.　　_____

October 13, 2004   **WOCOMAL Varsity Meet**

TEAM ROUND   **School: _____**

Team #:_____

Team Members:

   1. _____

   2. _____

   3. _____

   4. _____

   5. _____

Total Points for Team Round: _____